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Constant-Cutoff Approach to Magnetic Moments of Hyperons

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We calculate the magnetic moments of hyperons in the simplified CHK model, with the stabilizing term proportional to e^{-2} omitted, based on the constant-cutoff limit of the cutoff quantization method developed by Balakrishna, Sanyuk, Schechter, and Subbaraman, which avoids the difficulties with the usual soliton boundary conditions pointed out by Iwasaki and Ohyama. Thus we show that there is qualitative agreement with the experimental values and the accuracy is similar to that obtained with the complete CHK model.

1. INTRODUCTION

It was shown by Skyrme (1961, 1962) that baryons can be treated as solitons of a nonlinear chiral theory. The original Lagrangian of the chiral $SU(2)$ σ-model is

$$\mathcal{L} = \frac{F_\pi^2}{16} \operatorname{Tr} \partial_\mu U \, \partial^\mu U^+ \tag{1.1}$$

where

$$U = \frac{2}{F_\pi} (\sigma + i\boldsymbol{\tau} \cdot \boldsymbol{\pi}) \tag{1.2}$$

is a unitary operator ($UU^+ = 1$) and F_π is the pion decay constant. In (1.2), $\sigma = \sigma(\mathbf{r})$ is a scalar meson field and $\boldsymbol{\pi} = \boldsymbol{\pi}(\mathbf{r})$ is the pion-isotriplet.

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The classical stability of the soliton solution to the chiral σ-model Lagrangian requires an additional ad hoc term, proposed by Skyrme (1961, 1962), to be added to (1.1)

$$\mathcal{L}_{\mathrm{Sk}} = \frac{1}{32e^2}\,\mathrm{Tr}[U^{+}\partial_\mu U,\ U^{+}\partial_\nu U]^2 \tag{1.3}$$

with a dimensionless parameter e and where $[A, B] = AB - BA$. It was shown by several authors that, after the collective quantization using the spherically symmetric ansatz

$$U_0(\mathbf{r}) = \exp[i\boldsymbol{\tau}\cdot\mathbf{r}_0 F(r)], \qquad \mathbf{r}_0 = \mathbf{r}/r \tag{1.4}$$

the chiral model, with both (1.1) and (1.3) included, gives good agreement with experiment for several important physical quantities [Adkins *et al.* (1983); see also Witten (1979, 1983a,b); for extensive lists of additional references see Holzwarth and Schwesinger (1986) and Nyman and Riska (1990)]. Thus it should be possible to derive the effective chiral Lagrangian, obtained as a sum of (1.1) and (1.3), from a more fundamental theory like QCD. On the other hand, it is not easy to generate a term like (1.3) and give a clear physical meaning to the dimensionless constant e in (1.3) using QCD.

Mignaco and Wulck (1989) (MW) indicated therefore the possibility to build a stable single-baryon ($n = 1$) quantum state in the simple chiral theory with the Skyrme stabilizing term (1.3) omitted.

However, as pointed out by Iwasaki and Ohyama (1989), the quantum stabilization method in the form proposed by Mignaco and Wulck (1989) is not correct since in the simple σ-model the conditions $F(0) = -\pi$ and $F(\infty) = 0$ cannot be satisfied simultaneously. In other words, if the condition $F(0) = -\pi$ is satisfied, then $F(\infty) \to -\pi/2$, and the chiral phase $F = F(r)$ with correct boundary conditions does not exist.

In Dalarsson (1991a), I suggested a method to resolve this difficulty by introducing a radial modification phase $\varphi = \varphi(r)$ in the ansatz (1.4) as follows:

$$U(\mathbf{r}) = \exp[i\boldsymbol{\tau}\cdot\mathbf{r}_0 F(r) + i\varphi(r)], \qquad \mathbf{r}_0 = \mathbf{r}/r \tag{1.5}$$

Such a method provides a stable chiral quantum soliton, but the resulting model is an entirely noncovariant chiral model, different from the original chiral σ-model.

In the present paper I use the constant-cutoff limit of the cutoff quantization method developed by Balakrishna *et al.* (1991; see also Jain *et al.*, 1989) to construct a stable chiral quantum soliton within the original chiral σ-model. Then I apply this method to calculate the magnetic moments of $SU(3)$-octet baryons and show that there is qualitative agreement with the experimental values.

2. CONSTANT-CUTOFF STABILIZATION

Substituting (1.4) into the action obtained using the Lagrangian density (1.1), we obtain the static energy of the chiral baryon

$$E_0 = \frac{\pi}{2} F_\pi^2 \int_{\epsilon(t)}^{\infty} dr \left[r^2 \left(\frac{dF}{dr} \right)^2 + 2 \sin^2 F \right] \tag{2.1}$$

In (2.1) we avoid the singularity of the profile function $F = F(r)$ at the origin by introducing the cutoff $\epsilon(t)$ at the lower boundary of the space interval $r \in [0, \infty]$, i.e., by working with the interval $r \in [\epsilon, \infty]$. The cutoff itself is introduced following Balakrishna *et al.* (1991) as a dynamic time-dependent variable.

From (2.1) we obtain the following differential equation for the profile function $F = F(r)$:

$$\frac{d}{dr}\left(r^2 \frac{dF}{dr} \right) = \sin 2F \tag{2.2}$$

with the boundary conditions $F(\epsilon) = -\pi$ and $F(\infty) = 0$, such that the correct soliton number is obtained. The profile function $F = F[r; \epsilon(t)]$ now depends implicitly on time t through $\epsilon(t)$. Thus in the nonlinear σ-model Lagrangian

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\partial_\mu U \, \partial^\mu U^+) \, d^3x \tag{2.3}$$

we use the ansätze

$$U(\mathbf{r}, t) = A(t) U_0(\mathbf{r}, t) A^+(t), \qquad U^+(\mathbf{r}, t) = A(t) U_0^+(\mathbf{r}, t) A^+(t) \tag{2.4}$$

where

$$U_0(\mathbf{r}, t) = \exp\{ i\boldsymbol{\tau} \cdot \mathbf{r}_0 F[r; \epsilon(t)] \} \tag{2.5}$$

The static part of the Lagrangian (2.3), i.e.,

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\nabla U \cdot \nabla U^+) \, d^3x = -E_0 \tag{2.6}$$

is equal to minus the energy E_0 given by (2.1). The kinetic part of the Lagrangian is obtained using (2.4) with (2.5) and it is equal to

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\partial_0 U \, \partial_0 U^+) \, d^3x = bx^2 \, \mathrm{Tr}(\partial_0 A \, \partial_0 A^+) + c[\dot{x}(t)]^2 \tag{2.7}$$

where

$$b = \frac{2\pi}{3} F_\pi^2 \int_1^{\infty} \sin^2 F \, y^2 \, dy, \qquad c = \frac{2\pi}{9} F_\pi^2 \int_1^{\infty} y^2 \left(\frac{dF}{dy} \right)^2 y^2 \, dy \tag{2.8}$$

with $x(t) = [\epsilon(t)]^{3/2}$ and $y = r/\epsilon$. On the other hand, the static energy functional (2.1) can be rewritten as

$$E_0 = ax^{2/3}, \qquad a = \frac{\pi}{2} F_\pi^2 \int_1^\infty \left[y^2 \left(\frac{dF}{dy} \right)^2 + 2 \sin^2 F \right] dy \tag{2.9}$$

Thus the total Lagrangian of the rotating soliton is given by

$$L = c\dot{x}^2 - ax^{2/3} + 2bx^2 \dot{\alpha}_\nu \dot{\alpha}^\nu \tag{2.10}$$

where $\mathrm{Tr}(\partial_0 A \; \partial_0 A^+) = 2\dot{\alpha}_\nu \dot{\alpha}^\nu$ and α_ν ($\nu = 0, 1, 2, 3$) are the collective coordinates. In the limit of a time-independent cutoff ($\dot{x} \to 0$) we can write

$$H = \frac{\partial L}{\partial \dot{\alpha}^\nu} \dot{\alpha}^\nu - L = ax^{2/3} + 2bx^2 \dot{\alpha}_\nu \dot{\alpha}^\nu = ax^{2/3} + \frac{1}{2bx^2} J(J + 1) \tag{2.11}$$

where $\langle \mathbf{J}^2 \rangle = J(J + 1)$ is the eigenvalue of the square of the soliton laboratory angular momentum. A minimum of (2.11) with respect to the parameter x is reached at

$$x = \left[\frac{2}{3} \frac{ab}{J(J + 1)} \right]^{-3/8} \Rightarrow \epsilon^{-1} = \left[\frac{2}{3} \frac{ab}{J(J + 1)} \right]^{1/4} \tag{2.12}$$

The energy obtained by substituting (2.12) into (2.11) is given by

$$E = \frac{4}{3} \left[\frac{3}{2} \frac{a^3}{b} J(J + 1) \right]^{1/4} \tag{2.13}$$

This result is identical to the result obtained by Mignaco and Wulck, which is easily seen if we rescale the integrals a and b in such a way that $a \to (\pi/4)F_\pi^2 a$ and $b \to (\pi/4)F_\pi^2 b$ and introduce $f_\pi = 2^{-3/2} F_\pi$. However, in the present approach, as shown in Balakrishna *et al.* (1991), there is a profile function $F = F(y)$ with proper soliton boundary conditions $F(1) = -\pi$ and $F(\infty) = 0$ and the integrals a, b, and c in (2.8)–(2.9) exist and are shown in Balakrishna *et al.* (1991) to be $a = 0.78\ \mathrm{GeV}^2$, $b = 0.91\ \mathrm{GeV}^2$, and $c = 1.46\ \mathrm{GeV}^2$ for $F_\pi = 186$ MeV.

Using (2.13), we obtain the same prediction for the mass ratio of the lowest states as Mignaco and Wulck (1989), which agrees rather well with the empirical mass ratio for the Δ-resonance and the nucleon. Furthermore, using the calculated values for the integrals a and b, we obtain the nucleon mass $M(N) = 1167$ MeV, which is about 25% higher than the empirical value of 939 MeV. However, if we choose the pion decay constant equal to $F_\pi = 150$ MeV, we obtain $a = 0.507\ \mathrm{GeV}^2$ and $b = 0.592\ \mathrm{GeV}^2$, giving exact agreement with the empirical nucleon mass.

At this point it is of interest to know how large the constant cutoffs are for the above values of the pion decay constant in order to check if they are physically acceptable. Using (2.12), it is easily shown that for the nucleons ($J = 1/2$) the cutoffs are equal to

$$\epsilon = \begin{cases} 0.22 \text{ fm} & \text{for} \quad F_\pi = 186 \text{ MeV} \\ 0.27 \text{ fm} & \text{for} \quad F_\pi = 150 \text{ MeV} \end{cases} \tag{2.14}$$

From (2.14) we see that the cutoffs are too small to agree with the size of the nucleon (0.72 fm), as we should expect, since the cutoffs rather indicate the size of the quark-dominated bag in the center of the nucleon. Thus we find that the cutoffs are of reasonable physical size. Since the cutoff is proportional to F_π^{-1}, we see that the pion decay constant must be less than 57 MeV in order to obtain a cutoff which exceeds the size of the nucleon. Such values of the pion decay constant are not relevant to any physical phenomena.

3. THE REVIEW OF THE SIMPLIFIED CHK MODEL

Callan and Klebanov (1985) showed that a good description of the hyperon spectrum in the Skyrme (1961, 1962) model is obtained if the hyperons are treated as bound kaon–soliton systems. Callan, Hornbostel, and Klebanov (1988) (CHK) successfully completed this program. The basic idea of their model is to treat strangeness separately from isospin in the Skyrme model, assuming that the vacuum is approximately $SU(3)$-symmetric i.e., $F_K \approx F_\pi$. The strange baryons are generated by binding kaons in the field of "rotating" $SU(2)$ solitons. Since there is no static field associated with the strangeness number, it is essential in this picture that there exist bound states in the kaon–soliton complex giving rise to hyperons. CHK showed that such bound states exist. A remarkable property of the kaons in this model is that after quantization they look like s-quarks, due to topological effects. This leads to a spectroscopy of hyperons quite similar to that of quark models.

In the CHK approach the kaon–soliton field is written in the form

$$U = U_\pi^{1/2} U_K U_\pi^{1/2} \tag{3.1}$$

where U_π is an $SU(3)$ extension of the usual $SU(2)$ skyrmion field used to describe the nucleon spectrum, and U_K is the field describing the kaons,

$$U_\pi = \exp(2iF_\pi^{-1}\lambda_j \pi^j), \qquad j = 1, 2, 3 \tag{3.2}$$

$$U_K = \exp(2iF_\pi^{-1}\lambda_a K^a), \qquad a = 4, 5, 6, 7 \tag{3.3}$$

The λ matrices are the familiar $SU(3)$ matrices.

The Lagrangian density for a bound kaon–soliton system in the simplified Skyrme model, with the Skyrme stabilizing term (1.3) omitted, is given by

$$\mathcal{L} = \frac{F_\pi^2}{16} \operatorname{Tr} \partial_\mu U \, \partial^\mu U^+ + \frac{F_\pi^2}{48} (m_\pi^2 + 2m_K^2) \operatorname{Tr}(U + U^+ - 2)$$
$$+ \frac{\sqrt{3}}{24} F_\pi^2 (m_\pi^2 - m_K^2) \operatorname{Tr} \lambda_8 (U + U^+) \tag{3.4}$$

where m_π and m_K are pion and kaon masses, respectively, and

$$U_\pi = \begin{bmatrix} u_\pi & 0 \\ 0 & 1 \end{bmatrix}, \qquad U_K = \exp\left\{ i \frac{2^{3/2}}{F_\pi} \begin{bmatrix} 0 & K \\ K^+ & 0 \end{bmatrix} \right\} \tag{3.5}$$

In (3.5), u_π is the usual $SU(2)$-skyrmion field, given by (1.4), and $F = F(r)$ is a radial function which, for $m_\pi = 0$, satisfies the differential equation (2.2). The two-dimensional vector K in (3.5) is the kaon doublet

$$K = \begin{bmatrix} K^+ \\ K^0 \end{bmatrix}, \qquad K^+ = [K^- \ \overline{K}^0] \tag{3.6}$$

In addition to the simplified Skyrme model action obtained using the Lagrangian density (3.4), the Wess–Zumino action in the form

$$S = -\frac{iN_c}{240\pi^2} \int d^5x \, e^{\mu\nu\alpha\beta\gamma} \operatorname{Tr}(U^+\partial_\mu U \, U^+\partial_\nu U \, U^+\partial_\alpha U \, U^+\partial_\beta U \, U^+\partial_\gamma U) \tag{3.7}$$

must be included into the total action of a kaon–soliton system. In (3.7), N_c is the number of colors in the underlying QCD.

We now substitute (3.1), with U_π and U_K defined by (3.5), into the total action of the kaon–soliton system, expand U_K to second order in the kaon fields (3.6), and decompose the kaon fields (3.6) into modes with strangeness number $S = \pm 1$ as follows:

$$K = \sum_m [\overline{K}_m(\mathbf{r}) e^{i\omega_m t} \hat{b}_m^+ + K_m(\mathbf{r}) e^{-i\omega_m t} \hat{a}_m] \tag{3.8}$$

with $\hat{a}_m$ and $\hat{b}_m^+$ the annihilation and creation operators for $S = -1$ and $S = +1$ modes, respectively. Then we expand the kaon wave function $K_m(r)$ of $S = -1$ mode in terms of vector spherical harmonics

$$K(\mathbf{r}) = \sum_{\alpha, L} k_{\alpha L}(r) Y_{\alpha IL} \tag{3.9}$$

to obtain the one-dimensional Lagrangian density for $S = -1$ modes in the following form:

$$\mathcal{L} = \dot{k}^+ \dot{k} + \frac{dk^+}{dr} \frac{dk}{dr} + i\lambda(r)(k^+ \dot{k} - \dot{k}^+ k) - k^+[m_K^2 + V_{\text{eff}}(r)]k \tag{3.10}$$

where

$$v_{\text{eff}}(r) = -\frac{1}{4}\left[\left(\frac{dF}{dr}\right)^2 + 2\,\frac{\sin^2 F}{r^2}\right] + \frac{2}{r^2}\cos^4\frac{F}{2} \tag{3.11}$$

$$\lambda(r) = -\frac{N_c}{2\pi^2 F_\pi^2}\,\frac{\sin^2 F}{r^2}\,\frac{dF}{dr} \tag{3.12}$$

The Hamiltonian density corresponding to the Lagrangian density (3.10) is given by

$$\mathcal{H} = \Pi^+\Pi - \frac{dk^+}{dr}\frac{dk}{dr} - i\lambda(r)(k^+\Pi - \Pi^+k) + k^+[m_K^2 + V_{\text{eff}}(r)]k$$
$$+ k^+\lambda^2(r)k \tag{3.13}$$

From (3.13) we obtain the radial wave equation for the lowest bound-state antikaon wave function $u_0 = rk_p(r)$ in the form

$$\frac{d^2u_0}{dr^2} - v_{\text{eff}}(r)u_0 + [\omega^2 - m_K^2 + 2\omega\lambda(r)]u_0 = 0 \tag{3.14}$$

where ω is the lowest bound-state energy, and the antikaon modes are normalized according to

$$8\pi\int_\epsilon^\infty dr\, r^2[\omega + \lambda(r)]k_p^*(r)k_p(r) = 1 \tag{3.15}$$

In order to find the rotational modes of the kaon–soliton system we rotate the kaon and soliton fields as follows:

$$K \to a(t)K \tag{3.16}$$
$$U \to A(t)UA^+(t)$$

where

$$A(t) = \begin{bmatrix} a(t) & 0 \\ 0 & 1 \end{bmatrix} \tag{3.17}$$

is an $SU(2)$ subgroup of $SU(3)$. The angular momentum operator of the rotating soliton $\mathbf{I}$ is given by

$$\mathbf{I} = -i\Omega\,\mathrm{Tr}(A^+\partial_0 A\,\boldsymbol{\tau}) \tag{3.18}$$

where Ω is the moment of inertia of the soliton. The total angular momentum operator $\mathbf{J}$ is given by

$$\mathbf{J} = \mathbf{I} + \mathbf{T}$$

$$\mathbf{T} = \mathbf{L} + \frac{1}{2}(\hat{a}_i^+\tau_{ij}\hat{a}_j - \hat{b}_i\boldsymbol{\tau}_{ij}\hat{b}_j^+) = \mathbf{L} + \mathbf{J}^+ + \mathbf{J}^- \tag{3.19}$$

where $\mathbf{T}$ is the total angular momentum of kaons and antikaons, whereas $\mathbf{J}^+$ and $\mathbf{J}^-$ are spins of $S = +1$ and $S = -1$ modes, respectively. The strangeness operator is given by

$$\hat{S} = \hat{a}_i^+ \hat{a}_i - \hat{b}_i \hat{b}_i^+ \tag{3.20}$$

After the quantization the Hamiltonian for a baryon with N_- bound antikaons in the P state ($L = 1$) becomes

$$\begin{aligned} H &= E_0 + \omega N_- + \frac{1}{2\Omega} (\mathbf{I} + c\mathbf{T})^2 \\ &= E_0 + \omega N_- + \frac{1}{2\Omega} [c\mathbf{J}^2 + (1 - c)\mathbf{I}^2 + c(c - 1)\mathbf{T}^2] \end{aligned} \tag{3.21}$$

where

$$\Omega = \frac{2\pi}{3} F_\pi^2 \int_\epsilon^\infty dr\, r^2 \sin^2 F \tag{3.22}$$

$$c = 1 - \frac{8}{3} \omega \int_\epsilon^\infty dr\, r^2 k_p^*(r) \cos^2 \frac{F}{2} k_p(r) \tag{3.23}$$

The eigenvalue of the kaon angular momentum $\mathbf{T}$ is related to the strangeness as $T = |S|/2$. Introducing the total angular momentum

$$\mathbf{J} = \mathbf{I} + \mathbf{T} \tag{3.24}$$

we obtain the total energy of the kaon–soliton system:

$$\begin{aligned} E = E_0 + \omega|S| + \frac{1}{2\Omega} [cJ(J + 1) + (1 - c)I(I + 1) \\ + \frac{1}{4} c(c - 1)|S|(|S| + 2)] \end{aligned} \tag{3.25}$$

Using the constant-cutoff stabilization method, we obtain the following spectrum of hyperons in the simplified CHK model:

$$\begin{aligned} E = \omega|S| + \frac{4}{3} \Bigg\{ \frac{3}{2} \frac{a^3}{b} \Bigg[cJ(J + 1) + (1 - c)I(I + 1) \\ + \frac{1}{4} c(c - 1)|S|(|S| + 2) \Bigg] \Bigg\}^{1/4} \end{aligned} \tag{3.26}$$

and the following expression for the inertia of the soliton:

$$\Omega = b\left\{\frac{3}{2}\frac{1}{ab}\left[cJ(J+1) + (1-c)I(I+1) + \frac{1}{4}c(c-1)|S|(|S|+2)\right]\right\}^{3/4} \tag{3.27}$$

with a and b defined by (2.9) and (2.8), respectively.

4. THE ELECTROMAGNETIC CURRENT

The electromagnetic current J_μ is obtained from the vector current $V_{a\mu}$ ($a = 1, \ldots, 8$) as follows:

$$J_\mu = V_{3\mu} + 3^{-1/2}V_{8\mu} \tag{4.1}$$

The vector current $V_{a\mu}$ is obtained as the Noether current associated with the symmetry of the total action with respect to the transformation

$$U \to \exp\left(\frac{1}{2}\, i\epsilon^a\lambda_a\right) U \exp\left(-\frac{1}{2}\, i\epsilon^a\lambda_a\right) \tag{4.2}$$

where ϵ^a ($a = 1, \ldots, 8$) is the set of eight infinitesimally small Noether parameters. As $\epsilon^a \to 0$, we obtain from (4.2)

$$U \to U + i\epsilon^a[\lambda_a/2, U] = U + \epsilon^a\delta U_a \tag{4.3}$$

where $[A, B] = AB - BA$. The Noether current associated with the transformation (4.3) is

$$V_{a\mu} = 2\,\mathrm{Tr}\left[\frac{\delta S}{\delta(\partial^\mu U)}\,\delta U_a\right] = 2i\,\mathrm{Tr}\left\{\frac{\delta S}{\delta(\partial^\mu U)}\left[\frac{\lambda_a}{2}, U\right]\right\} \tag{4.4}$$

where S is the total action of the simplified CHK model. Thus we obtain

$$V_{a\mu} = -i\,\frac{F_\pi^2}{16}\,\mathrm{Tr}(\lambda_a U^+\partial_\mu U + \lambda_a U\partial_\mu U^+) + \frac{N_c}{96\pi^2}\,\epsilon_{\mu\nu\rho\sigma}\,\mathrm{Tr}(\lambda_a U^+\partial^\nu U U^+\partial^\rho U U^+\partial^\sigma U - \lambda_a U\partial^\nu U^+ U\partial^\rho U^+ U\partial^\sigma U^+) \tag{4.5}$$

Substituting (4.5) into (4.1), we obtain the expression for the electromagnetic current.

5. THE MAGNETIC MOMENTS

The spatial part of the electromagnetic current defined by (4.1) with (4.5) can be written as the sum of a transverse and a longitudinal component with respect to the direction $\mathbf{r}_0 = \mathbf{r}/r$. The magnetic moment $\mathbf{m}$ on the other hand is defined by the expression

$$\mathbf{J}_{tr} = \mathbf{m} \times \mathbf{r} \tag{5.1}$$

and it is determined by the transverse component of the electromagnetic current only.

Using (4.1) and (4.5), we obtain the isoscalar ($I = 0$) contribution to the magnetic moment operator in the form

$$m^3_{I=0}(r) = -i\,\frac{\mathrm{Tr}(A^+\partial_0 A\tau^3)}{4\pi^2}\,\frac{-\sin^2 F}{r^2}\,\frac{dF}{dr} + F^2_\pi\,\frac{\hat{a}^+_i\tau^3_{ij}\hat{a}_j + \hat{b}_i\tau^3_{ij}\hat{b}^+_j}{2r^2}\,k^*_p(r)k_p(r)\cos^2\frac{F}{2} \tag{5.2}$$

and the isovector contribution ($I = 1$)

$$m^3_{I=1}(r) = F^2_\pi\,\frac{-\mathrm{Tr}(A^+\tau^3 A\tau^3)}{8r^2}\left[\sin^2 F + (\hat{a}^+_i\hat{a}_i + \hat{b}_i\hat{b}^+_i)k^*_p(r)k_p(r)\cos^2\frac{F}{2}\left(1 - 4\sin^2\frac{F}{2}\right)\right] \tag{5.3}$$

In order to calculate the magnetic moments, we need to express the operators occurring in (5.2) and (5.3) in terms of the angular momentum operators, whose expectation values between baryon states are easily calculated. The operators occurring in the isoscalar term ($I = 0$) can be written as

$$-i\,\mathrm{Tr}(A^+\partial_0 A\tau^3) = \frac{1}{\Omega}\,I^3 \tag{5.4}$$

$$\frac{1}{2}\,(\hat{a}^+_i\tau^3_{ij}\hat{a}_j + \hat{b}_i\tau^3_{ij}\hat{b}^+_j) = J^{+3} - J^{-3} \tag{5.5}$$

while the operators occurring in the isovector term ($I = 1$) can be written as

$$-\mathrm{Tr}(A^+\tau^3 A\tau^3) = g(J_c, I)J^3_c I^3 \tag{5.6}$$

$$\hat{a}^+_i\hat{a}_i + \hat{b}_i\hat{b}^+_i = N_+ + N_- \tag{5.7}$$

where for a system consisting of a soliton and N_- bound antikaons we

Table I. Coefficients $g(J_c, I)$ in Soliton Quantum States

J_c	I	$g(J_c, I)$
0	0	0
1/2	1/2	8/3
1	1	1
3/2	3/2	8/15

introduce the collective spin operator $\mathbf{J}_c = \mathbf{I} - c\mathbf{J}^-$. The first few coefficients $g(J_c, I)$ in (5.6) are given in Table I.

The magnetic moment, in units of nuclear magneton $e/(2M_N)$, is defined by

$$\mu = \frac{2}{3} M_N \int d^3\mathbf{r}\ r^2 m^3(r) \tag{5.8}$$

For a system with a soliton and N_- bound antikaons the magnetic moment operator thus becomes

$$\mu = \mu_1 J_c^3 + (\mu_2 + \mu_3 N_-) g(J_c, I) J_c^3 I^3 + \mu_4 J^{-3} \tag{5.9}$$

where

$$\mu_1 = -\frac{2M_N}{3\pi\Omega} \int_\epsilon^\infty dr\ r^2 \sin^2 F \frac{dF}{dr} \tag{5.10}$$

$$\mu_2 = \frac{1}{2} M_N \Omega \tag{5.11}$$

$$\mu_3 = \frac{4\pi}{3} M_N F_\pi^2 \int_\epsilon^\infty dr\ r^2 k_p^*(r) k_p(r) \cos^2\frac{F}{2}\left(1 - 4\sin^2\frac{F}{2}\right) \tag{5.12}$$

$$\mu_4 = c\mu_1 - \frac{4\pi}{3} M_N F_\pi^2 \int_\epsilon^\infty dr\ r^2 k_p^*(r) k_p(r) \cos^2\frac{F}{2} \tag{5.13}$$

Introducing a dimensionless variable $y = r/\epsilon$, we obtain the following results for the quantities (5.10)–(5.13):

$$\mu_1 = \frac{2M_N}{3\pi^2 F_\pi^2 \beta} \epsilon^{-1} I_1 \tag{5.14}$$

$$\mu_2 = \frac{\pi}{8} M_N F_\pi^2 \epsilon^3 \beta \tag{5.15}$$

$$\mu_3 = \frac{32}{3\beta} I_3 \mu_2 \tag{5.16}$$

$$\mu_4 = c\mu_1 - \frac{32}{3\beta} I_4 \mu_2 \tag{5.17}$$

where α, β, I_1, I_3, and I_4 are dimensionless integrals given by

$$\alpha = 2\int_1^\infty \left[y^2\left(\frac{dF}{dy}\right)^2 + 2\sin^2 F \right] dy \tag{5.18}$$

$$\beta = \int_1^\infty dy\, \frac{8}{3} y^2 \sin^2 F(y) \tag{5.19}$$

$$I_1 = -\int_1^\infty dy\, y^2 \sin^2 F \frac{dF}{dy} \tag{5.20}$$

$$I_3 = \int_1^\infty dy\, y^2 k_p^*(y) k_p(y) \cos^2\frac{F}{2}\left(1 - 4\sin^2\frac{F}{2}\right) \tag{5.21}$$

$$I_4 = \int_1^\infty dy\, y^2 k_p^*(y) k_p(y) \cos^2\frac{F}{2} \tag{5.22}$$

The integrals (5.18)–(5.22) are universal for all baryons and in these $y = r/\epsilon$ is a dimensionless variable.

The results obtained so far agree with the previous studies (Nyman and Riska, 1990) in the limit $e \to \infty$. In the complete Skyrme model, with the Skyrme stabilizing term proportional to e^{-2} included, the quantities μ_i (i = 1, 2, 3, 4) are universal for all baryons. Furthermore, comparing (5.9) with the results for magnetic dipole moments of SU (3)-octet baryons given in Gasiorowicz and Rosner (1981), it is easily seen that μ_1, μ_2, and μ_4 are proportional to $\mu_u + \mu_d$, $\mu_u - \mu_d$, and μ_s, respectively, where μ_u, μ_d, and μ_s are magnetic moments of u-, d-, and s-quarks, respectively. The values of μ_i (i = 1, 2, 3, 4) that provide the best fit to the experimental data for the strange baryons and the exact agreement with the experimental data for the nucleons are given in Table II. With these values of μ_i (i = 1, 2, 3, 4) and using (5.9), we obtain the values of the magnetic moments of SU(3)-octet baryons given in Table II.

Using now the quantum stabilization method described in Dalarsson (1993) or here, we minimize the energy of the soliton with respect to the dimensional scale parameter ϵ. Thus quantities such as the moment of inertia Ω and the soliton energy E become functions of the angular momenta as in equations (3.26)–(3.27). Substituting these functions of angular momenta into (5.14)–(5.17), we obtain the following results for the quantities μ_i (i = 1, 2, 3, 4):

Table II. Fitted Magnetic Moments of $SU(3)$-Octet Baryons in the CHK Model[a]

Particle	μ	μ_{EXP}
p	(2.79)	2.793
n	(−1.90)	−1.901
Λ	−0.61	−0.613 ± 0.004
Σ^+	2.47	2.42 ± 0.05
Σ^0	0.79	—
Σ^-	−0.89	−1.157 ± 0.025
Ξ^0	−1.30	−1.250 ± 0.014
Ξ^-	−0.63	−0.69 ± 0.04

[a] $\mu_1 = 0.89$, $\mu_2 = 3.52$, $\mu_3 = -1.02$, $\mu_4 = -1.23$.

$$\mu_1 = \frac{2}{3\pi^2}\frac{M_N}{F_\pi} I_1 \left[\frac{2}{3}\left(\frac{\pi}{4}\right)^2 \frac{\alpha}{\beta^3}\right]^{1/4} \times [c\mathbf{J}^2 + (1 - c)\mathbf{I}^2 + c(c - 1)\mathbf{T}^2]^{-1/4} \tag{5.23}$$

$$\mu_2 = \frac{\pi}{8}\frac{M_N}{F_\pi}\beta\left[\frac{3}{2}\left(\frac{4}{\pi}\right)^2 \frac{1}{\alpha\beta}\right]^{3/4} [c\mathbf{J}^2 + (1 - c)\mathbf{I}^2 + c(c - 1)\mathbf{T}^2]^{3/4} \tag{5.24}$$

$$\mu_3 = \frac{32}{3\beta} I_3 \mu_2 \tag{5.25}$$

$$\mu_4 = c\mu_1 - \frac{32}{3\beta} I_4 \mu_2 \tag{5.26}$$

where α, β, I_1, I_3, and I_4 are dimensionless integrals given by (5.18)–(5.22) which remain universal for all baryons. However, we see from (5.23)–(5.26) that the coefficients μ_i ($i = 1, 2, 3, 4$) are no longer universal for all baryons, but vary between different baryon families. Using (5.9), we obtain the matrix elements of the magnetic moment operator for $SU(3)$-octet baryons as functions of coefficients μ_i ($i = 1, 2, 3, 4$), as shown in Table III.

The numerical calculation based on the fitted values $\mu_i(N)$ ($i = 1, 2, 3, 4$) for nucleons given in Table II gives the values for μ_i ($i = 1, 2, 3, 4$) for strange $SU(3)$-octet baryon families shown in Table IV.

Using the expressions given in Table III and the numerical results for coefficients μ_i ($i = 1, 2, 3, 4$) given in Table IV, we obtain the numerical results for magnetic moments of $SU(3)$-octet baryon states and compare these with the experimental values in Table V.

The calculated magnetic moments are in qualitative agreement with the experimental values, but the accuracy is not very satisfactory. The reason for this insufficient accuracy is the considerable simplification of the employed

Table III. Magnetic Moments of Baryons (for the Highest Spin State)

Particle	μ
p	$\frac{1}{2}\mu_1(N) + \frac{2}{3}\mu_2(N)$
n	$\frac{1}{2}\mu_1(N) - \frac{2}{3}\mu_2(N)$
Λ	$\frac{1}{2}\mu_4(\Lambda)$
Σ^+	$\frac{2}{3}\mu_1(\Sigma) + \frac{2}{3}\mu_2(\Sigma) + \frac{2}{3}\mu_3(\Sigma) - \frac{1}{6}\mu_4(\Sigma)$
Σ^0	$\frac{2}{3}\mu_1(\Sigma) - \frac{1}{6}\mu_4(\Sigma)$
Σ^-	$\frac{2}{3}\mu_1(\Sigma) - \frac{2}{3}\mu_2(\Sigma) - \frac{2}{3}\mu_3(\Sigma) - \frac{1}{6}\mu_4(\Sigma)$
Ξ^0	$-\frac{1}{6}\mu_1(\Xi) - \frac{2}{9}\mu_2(\Xi) - \frac{4}{9}\mu_3(\Xi) + \frac{2}{3}\mu_4(\Xi)$
Ξ^-	$-\frac{1}{6}\mu_1(\Xi) + \frac{2}{9}\mu_2(\Xi) + \frac{4}{9}\mu_3(\Xi) + \frac{2}{3}\mu_4(\Xi)$

Table IV. Coefficients μ_i (i = 1, 2, 3, 4) for Some Baryon Families

Particle	μ_1	μ_2	μ_3	μ_4
N (p, n)	0.89	3.52	−1.02	−1.23
Λ	0.89	3.48	−1.01	−1.21
Σ^+, Σ^0, Σ^-	0.83	4.30	−1.25	−1.73
Ξ^0, Ξ^-	0.84	4.24	−1.23	−1.67

Table V. Numerical Results for the Magnetic Moments of Baryons

Particle	μ	μ_{EXP}
p	(2.79)	2.793
n	(−1.90)	−1.901
Λ	−0.60	−0.613 ± 0.004
Σ^+	2.88	2.42 ± 0.05
Σ^0	0.84	—
Σ^-	−1.19	−1.157 ± 0.025
Ξ^0	−1.66	−1.250 ± 0.014
Ξ^-	−0.87	−0.69 ± 0.04

model, where only the soliton energy (and not the energy of the kaon–antikaon–soliton system as a whole) is minimized and the dynamical term in the rotational perturbation is only evaluated to first order.

6. CONCLUSIONS

The present paper shows the possibility of using the constant-cutoff approach to the CHK model for calculation of the magnetic moments of hyperons without the use of the Skyrme stabilizing term, proportional to e^{-2}, which makes the practical calculations very lengthy and painful. The results obtained so far are in qualitative agreement with the empirical values, but no attempt to calculate the magnetic moments of nucleons has been made. Furthermore, the second- and higher-order contributions to the dynamical term in the rotational perturbation are neglected. These aspects will be addressed in forthcoming numerical studies.

REFERENCES

Adkins, G. S., Nappi, C. R., and Witten, E. (1983). *Nuclear Physics B,* **228**, 552.
Balakrishna, B. S., Sanyuk, V., Schechter, J., and Subbaraman, A. (1991). *Physical Review D,* **45**, 344.
Bhaduri, R. K. (1988). *Models of the Nucleon,* Addison-Wesley, Reading, Massachusetts.
Callan, C. G., and Klebanov, I. (1985). *Nuclear Physics B,* **262**, 365.
Callan, C. G., Hornbostel, K., and Klebanov, I. (1988). *Physics Letters B,* **202**, 269.
Dalarsson, N. (1991a). *Modern Physics Letters A,* **6**, 2345.
Dalarsson, N. (1991b). *Nuclear Physics A,* **532**, 449.
Dalarsson, N. (1993). *Nuclear Physics A,* **554**, 580.
Gasiorowicz, S., and Rosner, J. L. (1981). *American Journal of Physics,* **49**, 954.
Holzwarth, G., and Schwesinger, B. (1986). *Reports on Progress in Physics,* **49**, 825.
Iwasaki, M., and Ohyama, H. (1989). *Physical Review,* **40**, 3125.
Jain, P., Schechter, J., and Sorkin, R. (1989). *Physical Review D,* **39**, 998.
Mignaco, J. A., and Wulck, S. (1989). *Physical Review Letters,* **62**, 1449.
Nyman, E. M., and Riska, D. O. (1990). *Reports on Progress in Physics,* **53**, 1137.
Skryme, T. H. R. (1961). *Proceedings of the Royal Society A,* **260**, 127.
Skyrme, T. H. R. (1962). *Nuclear Physics,* **31**, 556.
Witten, E. (1979). *Nuclear Physics B,* **160**, 57.
Witten, E. (1983a). *Nuclear Physics B,* **223**, 422.
Witten, E. (1983b). *Nuclear Physics B,* **223**, 433.